Electron acceleration by an intense short pulse laser in a static magnetic field in vacuum

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Electron acceleration by a laser pulse having Gaussian radial and temporal profiles of intensity has been studied in a static magnetic field in vacuum. The starting point of the magnetic field has been taken around the point where the peak of the pulse interacts with the electron and the direction of the static magnetic field is taken to be the same as that of the magnetic field of the laser pulse. The electron gains considerable energy and retains it in the form of cyclotron oscillations even after the passing of the laser pulse in the presence of an optimum static magnetic field. The optimum value of the magnetic field decreases with laser intensity and initial electron energy. The energy gain also depends upon the laser spot size and peaks for a suitable value. The energy gained by the electron increases with laser intensity and initial electron energy. The electron trajectory and energy gain for different parameters such as laser intensity, initial electron energy, and laser spot size have been presented.

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I. INTRODUCTION

With the development of the chirped-pulse-amplification technique, table-top high-peak power lasers have been successfully developed with light intensities as high as $I\lambda^2 \cong 10^{21}\ \mathrm{W/cm^2}$, where I and λ are the laser intensity and wavelength in units of $\mathrm{W/cm^2}$ and μm, respectively. Consequently, there have emerged many new frontier research areas in both applied and fundamental physics [1]. Among these, the development of a laser-driven electron acceleration mechanism is a fast advancing area of scientific research [2,3]. The early schemes envisaged the production of a large-amplitude plasma wave in different accelerators such as the plasma wake-field accelerator [4], the plasma beat-wave accelerator [5], the laser wake-field accelerator (LWFA) [5,6], the resonant laser plasma accelerator [7], and the self-modulated laser wake-field accelerator (SMLWFA) [8–10]. A major limitation of these schemes arises from the wave particle dephasing. One may overcome it by a surfatron concept [11] in which a traverse static magnetic field is superimposed on the plasma wave. Dieckmann *et al.* [12] examined electron acceleration by an electrostatic plasma wave in magnetized plasma. They assumed an electric field of the electrostatic wave $\mathbf{E}(x,t)=E_0(t)\sin(kx-\omega t)\hat{\mathbf{x}}$ propagating transverse to a uniform magnetic field $\mathbf{B}=B_0\hat{\mathbf{z}}$. The phase space distribution of electrons $f(x,v_x,v_y)$ at different time scales was recorded. Singh *et al.* [13] have shown that the duration of the interaction between plasma waves and electrons increases due to the self-generated azimuthal magnetic field and the electrons gain more energy.

Direct laser electron acceleration in vacuum has received attention in recent years [14–17]. Vacuum as a medium for electron acceleration has some advantages over plasma. The problems inherent in the laser-plasma interaction such as instabilities are absent in a vacuum. The group velocity of the laser pulse is higher in a vacuum than that in a plasma. This increases the duration of the interaction between the laser pulse and electron, thus increasing the energy gain. Furthermore, the peak energy attained by the electron increases with initial electron energy and this makes it easier to inject pre-accelerated electrons in a vacuum than that in a plasma. It is well known that a planar-laser pulse cannot be used for electron acceleration, since when it overtakes an electron the latter is ponderomotively driven forwards in the rising part, but then backwards in the trailing part, resulting in no net energy gain by the electron [18]. An electron can gain and retain significant energy if a wiggler magnetic field of suitable magnitude and period is externally applied. However, one of the fundamental limitations of this acceleration scheme is the dephasing of the trapped electron with respect to the driver laser wave [19]. Resonance can be maintained for longer duration for a suitably tapered wiggler period and the electron can gain much higher energy [20]. The required wiggler period increases with initial electron energy. In this paper it is shown that if an optimum static magnetic field in the same direction as the magnetic field of the laser pulse is externally applied during the trailing part of the pulse, the electron can gain and retain significant energy in the form of cyclotron oscillations even after the passing of the laser pulse.

In this paper, results of a relativistic two-dimensional single-particle code for electron acceleration by an intense short pulse laser in the presence of a static magnetic field parallel to a magnetic field of the laser pulse have been presented. Relativistic equations for electron acceleration have been formulated in the next section. Numerical results are presented in Sec. III. Finally, a brief discussion of the results is given in Sec. IV.

II. RELATIVISTIC ANALYSIS

Consider the propagation of a laser pulse with the vector potential

$$\mathbf{A}_L = -\hat{x}A_0 \sin(\omega t - kz)\exp\{-[t-(z-z_L)/c]^2/\tau^2\} \times \exp[-r^2/2r_0^2], \qquad (1)$$

where z_L is the initial position of pulse peak, τ is the pulse

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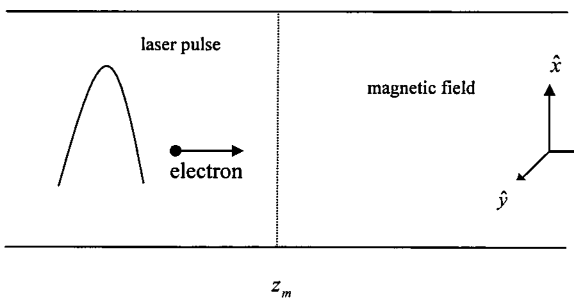

FIG. 1. Schematic of electron acceleration by an intense short pulse laser in a static magnetic field in the downwards $(-\hat{y})$ direction.

duration, $r^2=x^2+y^2$, and r_0 is the laser spot size.

The electromagnetic fields related to the vector potential of the laser pulse are

$$\mathbf{E}_L = -\frac{\partial \mathbf{A}_L}{\partial t}, \quad \mathbf{B}_L = \nabla \times \mathbf{A}_L. \tag{2}$$

There also exists an external magnetic field

$$\mathbf{B} = -\hat{y}B_0. \tag{3}$$

Figure 1 shows a schematic of the electron acceleration by an intense short pulse laser in a static magnetic field.

The equations governing electron momentum and energy are

$$\frac{dP_x}{dt} = e\left[\frac{\partial A_L}{\partial t} + v_z\left(\frac{\partial A_L}{\partial z} - B_0\right)\right], \tag{4}$$

$$P_y = 0, \tag{5}$$

$$\frac{dP_z}{dt} = -ev_x\left(\frac{\partial A_L}{\partial z} - B_0\right), \tag{6}$$

$$\frac{d\gamma}{dt} = \frac{ev_x}{m_0c^2}\frac{\partial A_L}{\partial t}, \tag{7}$$

where $-e$ and m_0 are electron charge and rest mass, respectively.

Equations (6) and (7) yield

$$\frac{d}{dt}(P_z - \gamma m_0 c - xeB_0) = 0,$$
$$P_z - \gamma m_0 c - xeB_0 = C_1, \tag{8}$$

where C_1 is given by the initial conditions. If the electron is at the origin with $P_x=0$ initially and $\gamma=\gamma_0,$ then $C_1=(\gamma_0^2-1)^{1/2}-\gamma_0 m_0 c$:

$$\frac{P_z}{m_0c} = (\gamma_0^2-1)^{1/2} - \gamma_0 + \gamma + xeB_0/(m_0c). \tag{9}$$

Using Eq. (9) with $\gamma^2=1+(P_x^2+P_z^2)/m_0^2,c^2$, we obtain

$$\frac{P_x}{m_0c} = \{\gamma^2 - 1 - [(\gamma_0^2-1)^{1/2} - \gamma_0 + \gamma + xeB_0/(m_0c)]^2\}^{1/2}. \tag{10}$$

In the following discussion, the following dimensionless variables will be used: $a_0=eA_0/m_0c$, $b_0=eB_0/m_0\omega$, $t\rightarrow\omega t$, $\tau\rightarrow\omega\tau$, $x\rightarrow\omega x/c$, $z\rightarrow\omega z/c$, $z_L\rightarrow\omega z_L/c$, $dx/dt \rightarrow(dx/dt)/c$, $dz/dt\rightarrow(dz/dt)/c$, $k\rightarrow kc/\omega$. In these variables, one can write Eqs. (7), (9), and (10) as follows:

$$\frac{dx}{dt} = \frac{1}{\gamma}\left[\gamma^2 - 1 - \{(\gamma_0^2-1)^{1/2} - \gamma_0 + \gamma + xb_0\}^2\right]^{1/2}, \tag{11}$$

$$\frac{dz}{dt} = \frac{1}{\gamma}\left[(\gamma_0^2-1)^{1/2} - \gamma_0 + \gamma + xb_0\right], \tag{12}$$

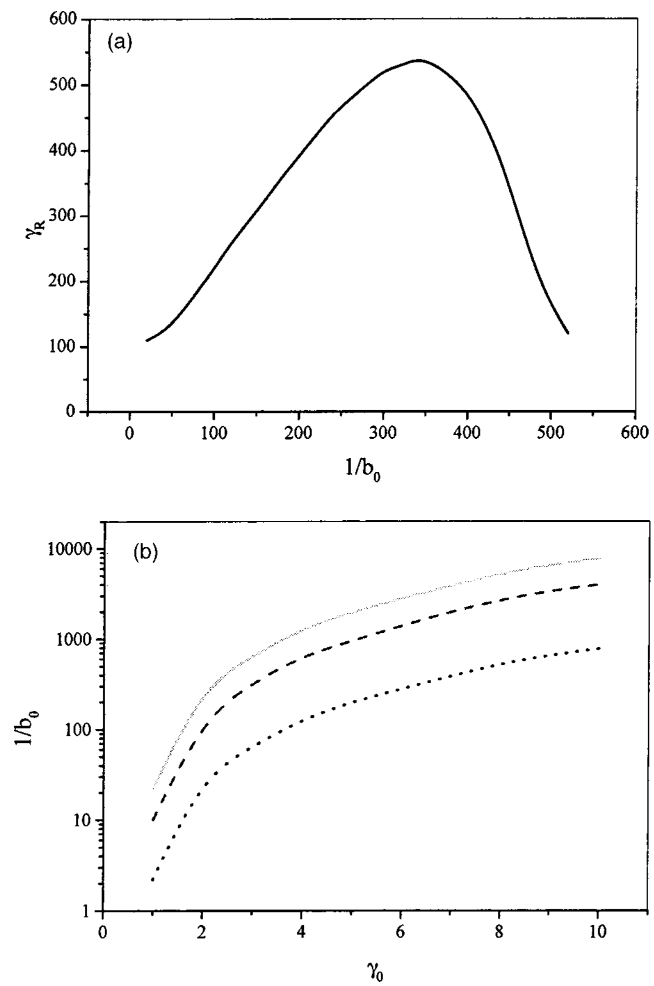

FIG. 2. (a) Retained energy γ_R as a function of $1/b_0$ for $a_0=5,\gamma_0=3$, $r_0=85$, and $z_m=3200$ and (b) $1/b_0$ as a function of initial electron energy for $a_0=1$, 5, and 10 (dotted, dashed, and solid lines, respectively).

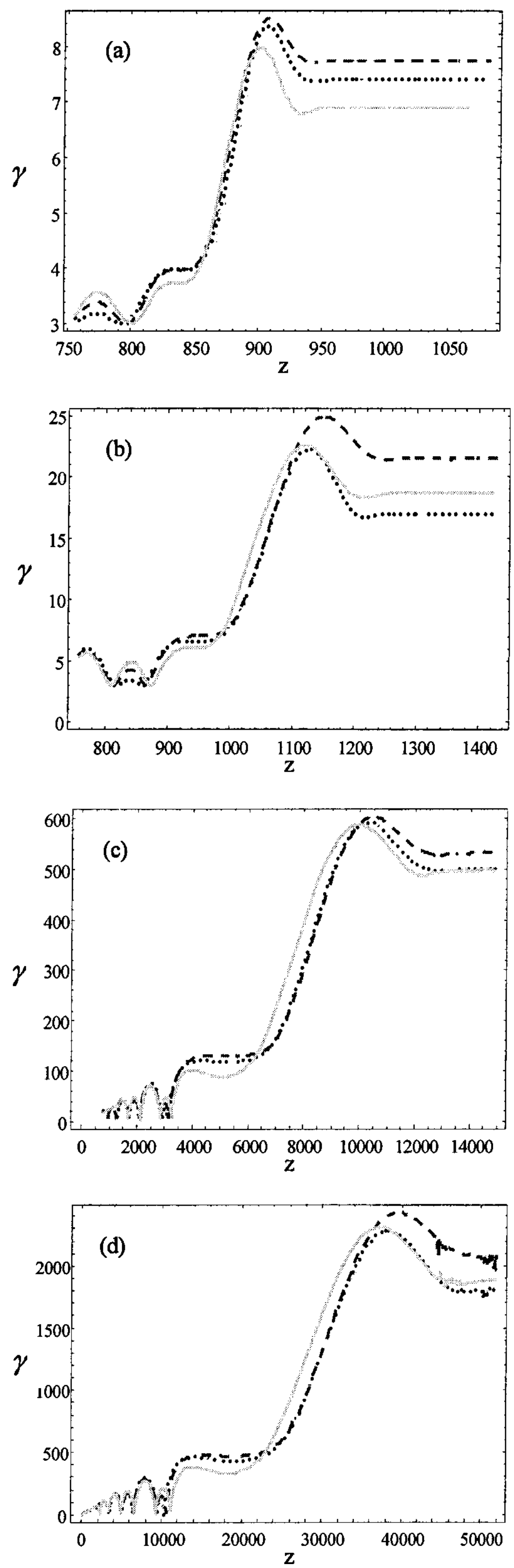

FIG. 3. Electron energy γ as a function of z at $\gamma_0=3$ for (a) $a_0=0.5$, $b_0=0.025$, $z_m=815$, and $r_0=7.8$, 9, and 12, (b) $a_0=1$, $b_0=0.015$, $z_m=875$, and $r_0=15,17$, and 20, (c) $a_0=5$, $b_0=0.003$, $z_m=3200$, and $r_0=80$, 85, and 95, and (d) $a_0=10$, $b_0=0.0015$, $z_m=11\,000$, and $r_0=161,175$, and 210. Dotted, dashed, and solid lines correspond to increasing order of spot sizes, respectively.

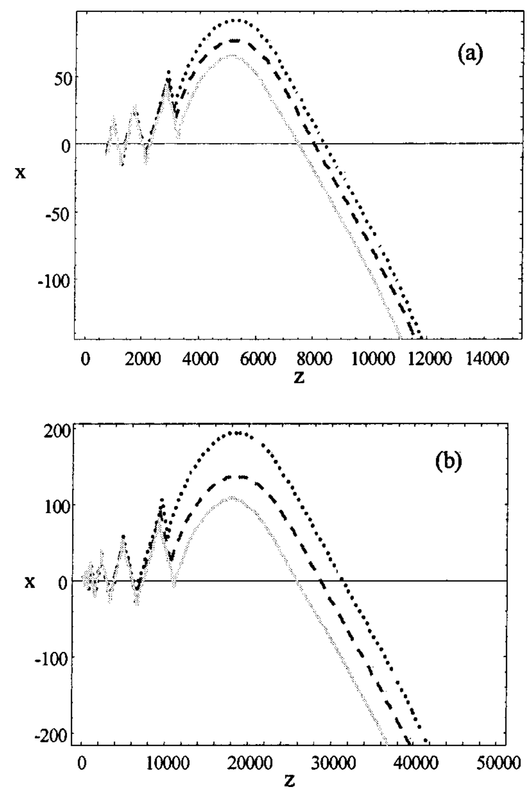

FIG. 4. (a) and (b) Electron trajectories in the x-z plane. The parameters are same as in Figs. 3(c) and 3(d), respectively.

$$\frac{d\gamma}{dt} = -a_0 \exp\left[-\frac{[t-(z-z_L)/c]^2}{\tau^2}\right]\exp\left[-\frac{r^2}{2r_0^2}\right]\frac{1}{\gamma} \times[\gamma^2-1-\{(\gamma_0^2-1)^{1/2}-\gamma_0+\gamma+xb_0\}^2]^{1/2} \times\left[\cos(t-z)-\frac{2(t-(z-z_L)/c)}{\tau^2}\sin(t-z)\right]. \quad (13)$$

Equations (11)–(13) are coupled ordinary differential equations of order 1. These have been solved numerically for electron trajectory and energy by the fourth-order Runge-Kutta method by assuming the initial electron position at the origin and initial electron energy γ_0.

III. NUMERICAL RESULTS

The electron energy γ as a function of z for different parameters has been obtained. The results are presented in the form of dimensionless variables. The values of the pulse length, magnetic field, etc., turn out different for different laser wavelengths. I have chosen $\tau=25$ and $z_L=-50$. The point z_m is chosen such that the electron interacts with the pulse in the presence of a static magnetic field during the trailing part. The results are as follows.

It is well known that the electron gains energy during the rising part of the laser pulse and loses it during the trailing part, resulting in a negligible net energy gain. However, if a suitable static magnetic field is present during the trailing part of the pulse, the electron gains considerable energy and retains it even after the passing of the pulse. The retained

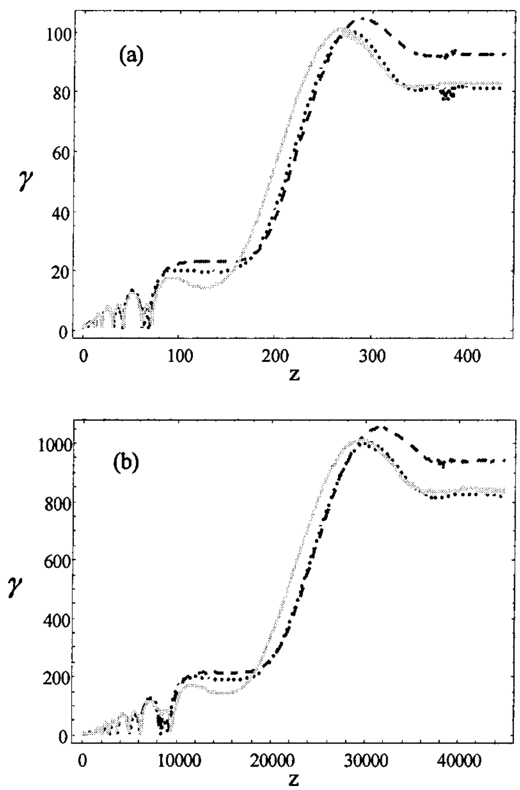

FIG. 5. Electron energy γ as a function of z at $a_0=5$ for (a) $\gamma_0=1$, $b_0=0.1$, $z_m=71$, and $r_0=13.6$, 15, and 16.3 and (b) $\gamma_0=5$, $b_0=0.001$, $z_m=9400$, and $r_0=137$, 150, and 167. Dotted, dashed, and solid lines correspond to increasing order of spot sizes, respectively.

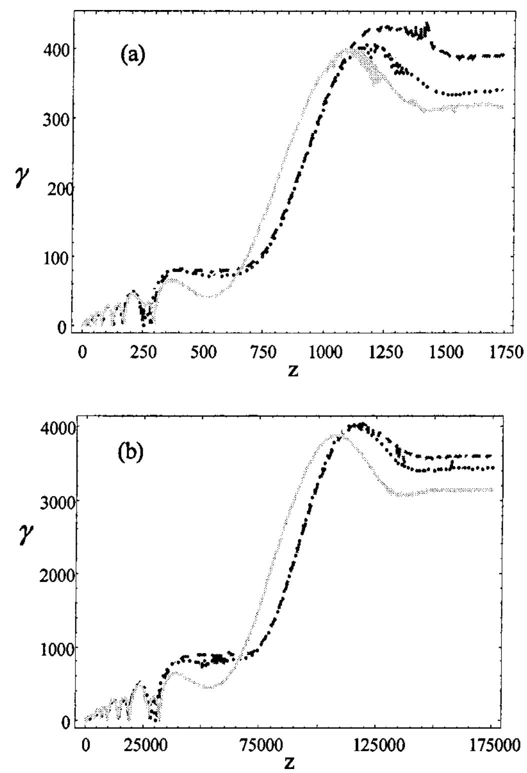

FIG. 6. Electron energy γ as a function of z at $a_0=10$ for (a) $\gamma_0=1$, $b_0=0.045$, $z_m=290$, and $r_0=28$, 32, and 34 and (b) $\gamma_0=5$, $b_0=0.0005$, $z_m=32\,000$, and $r_0=270$, 300, and 355. Dotted, dashed, and solid lines correspond to increasing order of spot sizes, respectively.

energy peaks for an optimum magnetic field. Figure 2(a) shows the variation of the retained energy as a function of normalized external magnetic field, and Fig. 2(b) shows the optimum value of the magnetic field as a function of initial electron energy for different laser intensities. The value of the optimum magnetic field decreases with laser intensity and initial electron energy. The results of Figs. 3–6 have been obtained for optimum values of the magnetic field.

The energy gained by the electron also depends upon the laser spot size and peaks for a suitable value. The results of Figs. 3–6 have been shown for three spot sizes; the middle one corresponds to a suitable spot size.

Figures 3(a)–3(d) show electron energy γ as a function of z for $a_0=0.5$, 1, 5, and 10, respectively, at $\gamma_0=3$. It can be seen that value of the suitable spot size and position (z_m) increases with laser intensity. Furthermore, it can also be seen that the energy gained by the electron increases with laser intensity; however, the quantity $(\gamma-\gamma_0)/a_0^2$ remains approximately the same for all cases, implying that the efficiency of energy gain with respect to laser intensity remains approximately the same. Using $B_0=2\pi m_0 cb_0/e\lambda$, we can calculate value of the optimum magnetic field in tesla. For a laser pulse with wavelength of 1 μm, the optimum value of the magnetic field corresponding to $b_0=0.025$, 0.015, 0.0030, and 0.0015 turns out to be nearly 250, 150, 30, and 15 T, respectively.

Figures 4(a) and 4(b) show electron trajectories in the x-z plane corresponding to the parameters of Figs. 3(c) and 3(d), respectively. The electron traverses more distance in the transverse direction for higher laser intensity. The electron trajectories in the other cases are similar to these and, therefore, have not been shown.

Figures 5 and 6 show the electron energy γ as a function of z for $a_0=5$ and $a_0=10$, respectively, for different initial electron energies ($\gamma_0=1$ and $\gamma_0=10$). It can be seen that the value of a suitable spot size and the position (z_m) at which the electron experiences the peak of the pulse intensity increases with initial electron energy. Furthermore, the energy gained by the electron increases with initial electron energy and the quantity $(\gamma-\gamma_0)/a_0^2$ is higher for $\gamma_0=10$ than that for $\gamma_0=1$ in both Figs. 5 and 6, implying that the efficiency of the energy gain with respect to the laser intensity increases with initial electron energy for a given laser intensity. For a laser pulse with wavelength of 1 μm, the optimum value of the magnetic field corresponding to $b_0=0.1$, 0.045, 0.001, and 0.0005 turns out nearly 1000, 450, 10, and 5 T, respectively.

IV. DISCUSSION

An electron gains energy during the rising part of the laser pulse and loses it during the trailing part, resulting in negligible energy gain. The presence of a suitable static magnetic field during the trailing part of the pulse leads not only to an enhancement of the energy gain but also to the retention of most of the energy in the form of cyclotron oscillations after the passing of the laser pulse. If the value of magnetic field is low, the electron experiences some deceleration, interacting with the trailing part of the pulse, and if it is too high, the electron escapes from the pulse; hence, the energy gained by the electron peaks for an optimum magnetic field. Similarly, for a small spot size the electron escapes from the pulse and for a large spot size the electron experiences some deceleration, interacting with the trailing part; therefore, the energy gain peaks for a suitable spot size. An electron traverses a larger longitudinal distance during the rising part of the pulse at higher laser intensity and initial electron energy; hence, the starting point of the static magnetic field has been chosen accordingly. The energy gain increases with initial electron energy because the duration of interaction between laser pulse increases with initial electron energy. The electron gains very high energy at high laser intensity and high initial energy; however, the required value of the spot size also increases. At low laser intensity and low initial electron energies, the optimum values of the magnetic field are very high; however, intense magnetic fields with duration several orders of magnitude longer than that of a short laser pulse are available [21]. If such a strong magnetic field is not available at a laboratory, then the experiment can be carried out at high initial electron energy and high laser intensity because the optimum value of the magnetic field decreases with laser intensity and initial electron energy.

[1] D. Umstadter, Phys. Plasmas **8**, 1774 (2001); J. Phys. D **36**, R151 (2003).
[2] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **24**, 252 (1996).
[3] A. Modena *et al.*, Nature (London) **377**, 606 (1995).
[4] P. Chen, Part. Accel. **20**, 171 (1985).
[5] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[6] L. M. Gorbunov and V. I. Kirsanov, Zh. Eksp. Teor. Fiz. **93**, 509 (1987).
[7] D. Umstadter, E. Esarey, and J. Kim, Phys. Rev. Lett. **72**, 1224 (1994).
[8] P. Sprangle, E. Esarey, J. Krall, and G. Joyce, Phys. Rev. Lett. **69**, 2200 (1992).
[9] T. M. Antonsen, Jr. and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).
[10] N. E. Andreev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **55**, 551 (1992).
[11] T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).
[12] M. E. Dieckmann, P. Ljung, A. Ynnerman, and K. G. McClements, IEEE Trans. Plasma Sci. **30**, 20 (2002).
[13] K. P. Singh, V. L. Gupta, Lalita Bhasin, and V. K. Tripathi, Phys. Plasmas **10**, 1493 (2003).
[14] G. Malka, E. Lefebvre, and J. L. Miquel, Phys. Rev. Lett. **78**, 3314 (1997).
[15] P. Mora and B. Quesnel, Phys. Rev. Lett. **80**, 1351 (1998).
[16] G. V. Stupakov and M. S. Zolotorev, Phys. Rev. Lett. **86**, 5274 (2001).
[17] P. X. Wang *et al.*, J. Appl. Phys. **91**, 856 (2002).
[18] Wei Yu, V. Bychenkov, Y. Sentoku, M. Y. Yu, Z. M. Sheng, and K. Mima, Phys. Rev. Lett. **85**, 570 (2000).
[19] F. V. Hertmann, *High-Field Electrodynamics* (CRC Press, Boca Raton, FL, 2001).
[20] K. P. Singh and V. K. Tripathi, Phys. Plasmas **2**, 743 (2004).
[21] http://www.lanl.gov/mst/nhmfl/magnets.shtml